Engineering Notes

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Wave Drag Prediction Using a Simplified Supersonic Area Rule

E.J. Jumper*
Air Force Institute of Technology
Wright-Patterson AFB, Ohio

Introduction

THE supersonic area rule, first reported by Jones,[1] is a commonly used method of determining wave drag that, as shown by Nelson and Welsh,[2] gives relatively good agreement with experiment. Many explanations of the supersonic area rule are misleading and the reader is directed to a report by Lomax[3] for the most detailed and lucid mathematical derivation of the rule as well as a clear statement of its limitations. The most widely used computer programs incorporating the method have their origins in a program written by the Boeing Company and reported by Harris.[4] While these programs give satisfactory results, they require a large amount of core and extensive input data describing the detailed geometry of the aircraft configuration to be analyzed. Experimental work, reported by Hall,[5] performed in conjunction with transonic-area-rule studies suggests that input this detailed may not be required in many cases.

The present investigation examined a modification to the supersonic area rule that greatly simplifies the input data and significantly reduces the complexity and core requirement of the associated computer program. The essence of the modification was to mathematically construct a simple equivalent body of revolution based on planes cut normal to the fuselage axis of the full aircraft configuration. The supersonic area rule was then applied to the axis-normal equivalent body of revolution, a modification not without precedent. Harris[4] described a similar technique for simplifying the description of the fuselage in the early Boeing program; however, nothing in the literature has suggested being so bold as to collapse the entire wing structure, tail structure, etc., into a single body of revolution for use in the supersonic area rule.

Theory

According to Jones,[1] the supersonic-area-rule equation for coefficient of wave drag C_{D_w} is given by

$$C_{D_w} = \frac{1}{2\pi}\int_0^{2\pi} C_{D_w}(\theta)\,d\theta \tag{1}$$

where $C_{D_w}(\theta)$ is

$$C_{D_w}(\theta) = -\frac{1}{2\pi S_b}\int_0^L\int_0^L S''(\xi)S''(x)\ln\left|\frac{x}{L}-\frac{\xi}{L}\right|d\xi\,dx \tag{2}$$

The S'' of Eq. (2) are second derivatives with respect to the axial position of the forward projection of the Mach plane, sectional area, where the Mach plane is inclined to the axis by the Mach angle μ, equal to $\sin^{-1}(1/M_\infty)$. In the usual application of the supersonic area rule the integration of Eq. (2) would be performed at all Mach plane orientations θ and the average taken via Eq. (1). In practice, Eq. (2) is numerically evaluated for 360 specific θ, 1 deg apart, or 180 if the aircraft configuration is symmetric. Further, new area distributions $S(x)$ must be determined at each θ. These area distributions are, in general, difficult to evaluate and may be further complicated because the planes may cut through those portions of the aircraft configuration that are disjoint.

According to the modification proposed here, an equivalent body of revolution is mathematically constructed by determining the axis-normal, cross-sectional area of the aircraft configuration as a function of axial position. Equation (2) is then numerically evaluated as in the unmodified method, where $S(x)$ is again the forward projection of the Mach plane, sectional area.

Several computational advantages of the modified over the unmodified supersonic area rule are immediately apparent. First, no matter how many Mach numbers are of interest, only one equivalent body involving the entire aircraft configuration need be determined; whereas in the unmodified rule at least 180 such "equivalent" bodies must be determined for each Mach number of interest. Second, the single equivalent body necessary for the modified rule is made with axis-normal planes; this amounts to a simple area distribution, which, if not already available, is easy to construct. Third, only a single forward projection of the Mach plane, sectional area distribution need be determined for each Mach number of interest because this distribution is constructed with respect to the equivalent body of revolution and, as such, $S(x)$ is independent of θ orientation. Fourth, again because of symmetry, only half the forward projection, sectional area need be evaluated, i.e., $0 \le \phi \le 180$ deg, where ϕ refers to the radius location of the forward projection of the Mach plane, sectional area (see Fig. 1a). Finally, the $C_{D_w}(\theta)$ found by a single integration of Eq. (2) is equal to C_{D_w} of Eq. (1).

Depending on the number of Mach numbers investigated, the number of computations is thus cut by approximately two orders of magnitude and the complexity of the algorithm is greatly reduced.

Numerical Evaluation of C_{D_w}

A computer program was written to numerically integrate Eq. (2) for a body of revolution by dividing the axis into a number of small segments of length Δx_i. The initial area distribution was read in as data. The $S(x)$ distribution was then determined for the particular Mach number of interest. It is possible to perform this operation for a body of revolution in two-dimensional space2 It can be shown that an angle ϕ for the Mach plane (see Fig. 1a) is related to γ in the two-dimensional plane (see Fig. 1b) by the following

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*Associate Professor, Department of Aeronautics and Astronautics. Member AIAA.

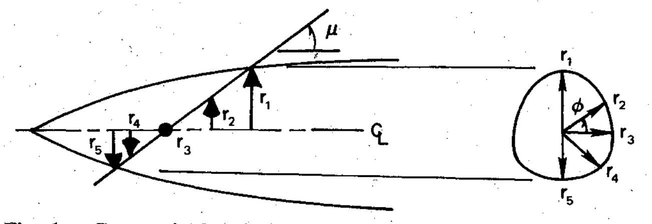

Fig. 1a Geometric description for computation of the radii of the new Mach plane body of revolution applied to a simple body of revolution.

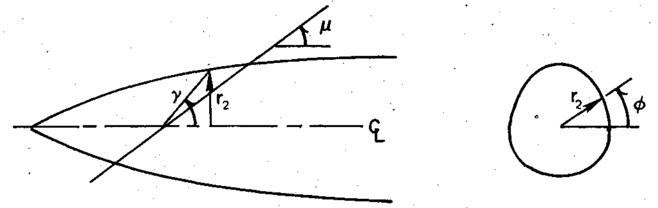

Fig. 1b Relation between ϕ and γ.

relationship:

$$\tan\gamma = \tan\mu / \sin\phi \tag{3}$$

By using the two-dimensional plane technique, ϕ was chosen at 1 deg increments through a total of π rad (180 deg) and the corresponding γ were used to determine the appropriate radii for determining half of the forward projection, Mach plane, sectional area. The total area was obtained by multiplying by two.

The intersection of the line at an angle γ and the radius of the body of revolution was determined by marching to the right (or left if ϕ was less than 0) in increments of Δx; computing the height of the y leg of the triangle determined by the γ angle ray, x marching distance, and the height y; and comparing the height with the radius at that location. Once the radius had been exceeded, an iterative process was used to let y approach the linear interpolated r to within some small predetermined amount.

Once $S(x)$ was determined for the particular Mach number, the numerical integration of Eq. (2) was carried out using the following simple algorithm:

$$C_{D_w} = -\frac{1}{2\pi S_b}\left\{\sum_{i=1}^{n} S_i''\Delta x_i\left[\sum_{j=1}^{i-1}\left(S_j''\ln\left|\frac{x_i}{L}-\frac{\xi_j}{L}\right|\Delta\xi_j\right) + 2S_i''L\left|\left(\frac{x_i}{L}-\frac{\xi_{i-1}}{L}\right)\ln\left(\frac{x_i}{L}-\frac{\xi_{i-1}}{L}\right)-\left(\frac{x_i}{L}-\frac{\xi_{j-1}}{L}\right)\right| + \sum_{j=I+1}^{n}\left(S_j''\ln\left|\frac{x_i}{L}-\frac{\xi_j}{L}\right|\Delta\xi_j\right)\right]\right\} \tag{4}$$

The summation was broken up and an analytic expression used at $i=j$ to avoid the singularity in the natural logarithm as x approaches ξ.

Equation (4) requires a second derivative of the area schedule at each axial location. Several schemes for dealing with this problem were tried, including fitting the area schedule with a cubic spline that advertised a match of the point and its first and second derivatives. The cubic spline, however, proved to be the source of large errors. A five-point, nested average of simple differences was found to be satisfactory for approximating the required derivatives. Starting with the area distribution $S(x)$, five slopes surrounding a given axis location were averaged to arrive at a first derivative value for that point. The same procedure was again used to approximate the second derivative from the first derivative values. Each axial location was handled in this manner except at the extreme ends of the body where a five-point nest was not possible. In these two regions, progressively smaller slope-averaging ranges were used.

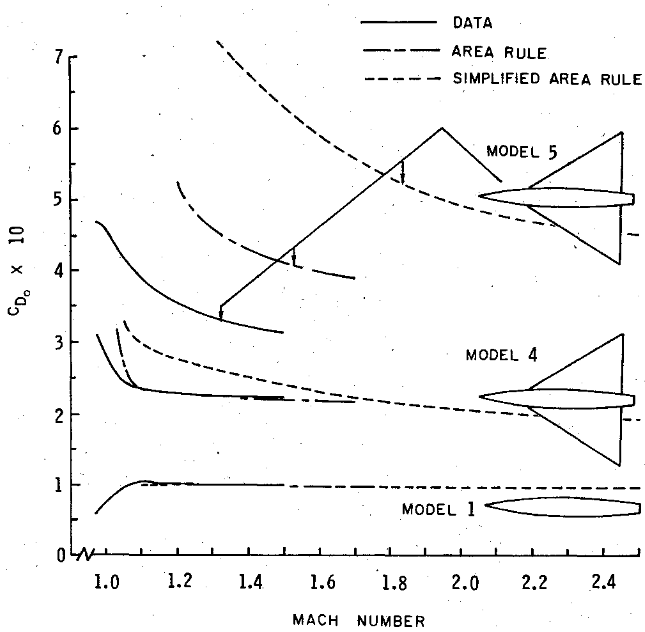

Fig. 2 Comparison of predicted wave drag at zero lift by the simplified supersonic area rule and the supersonic area rule with experimental data (drag coefficient based on the maximum cross-sectional area of the unwinged body of revolution).

Results

Since both the modified and unmodified supersonic area rules are identical for a body of revolution, the computer program was run on a number of control cases to compare the predicted drag to both experimental data and supersonic-area-rule results for bodies of revolution from Ref. 2. The computer program matched the results of Ref. 2 for these cases (see the curve for model 1 in Fig. 2, for example).

To test the validity of the simplification, two additional cases were run, those for models 4 and 5 of Ref. 2. These two cases were chosen because model 4 represented an example of moderately large areas located far off the fuselage axis for which the unmodified supersonic area rule gave good agreement to experiment, and model 5 represented an extreme case of large areas located far off the fuselage axis for which even the unmodified supersonic area rule began to fail. The results of these cases are also shown in Fig. 2. All three models are described in detail elsewhere.[6] The results for model 4 show that the unmodified supersonic area rule predicts the data very closely, while the simplified area rule is off by as much as 20% at $M=1.1$, but is within 7% by $M=1.5$. The results for model 5 show that even the unmodified area rule is unacceptable and the simplification is even worse.

The simplified supersonic area rule was then applied to a single case for which the intended use was to predict the drag-increment penalty of adding a near-axis protuberance to an existing aircraft configuration. The case chosen was that for an F-15 with and without two McDonnell-Douglas Fast Pack pallets in place. These configurations had been studied previously by Lemley et al.,[7] and both unmodified supersonic-area-rule predictions and wind-tunnel data existed.

Figure 3 shows the results of the drag-increment predictions for both the modified and unmodified supersonic area rule as well as the wind-tunnel data. The modified-rule results were obtained by first using the area distribution for the clean F-15 to obtain the coefficient of drag. Then the area distribution was modified to account for the additional area due to the conformal pallets (the insert in Fig. 3 gives the area distributions used for these calculations). The curve of Fig. 3 was then obtained by subtracting the without-pallet-configuration results from the with-pallet results. In this case the simplified supersonic area rule more closely predicted the data than did the area rule.

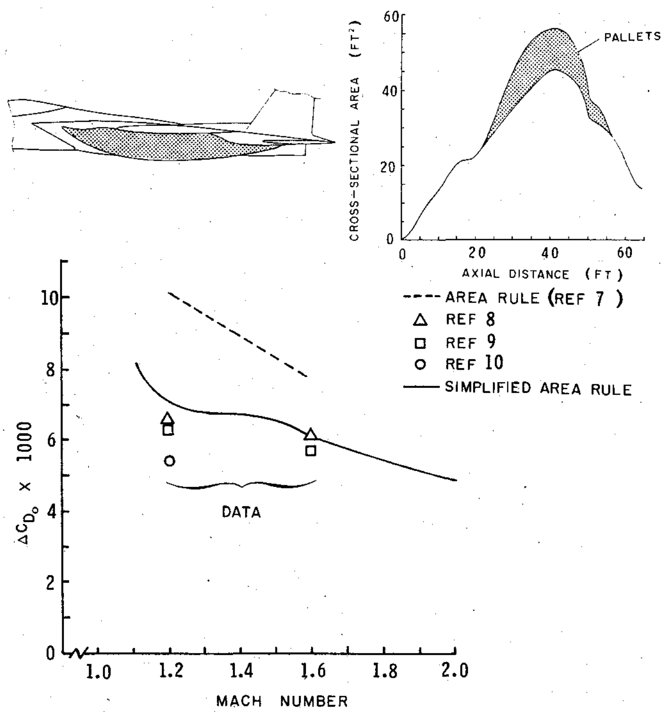

Fig. 3 Incremental drag due to adding the Fast Pack pallets [drag coefficient based on wing planform area (608 ft²)]; insert shows location and area distribution of F-15 Fast Pack pallets.

Finally, it should be noted that the program used for this study can easily fit in a 32K microcomputer and the computation times for each Mach number take less than 2 min for an axis with 100 axial grid locations.

Conclusions

The results of this study seem to indicate that the modified supersonic area rule yields relatively good predictions of the total wave drag for those cases where the unmodified area rule also yields good results. This result exceeds the expectations set for the envisioned use: to estimate drag increments due to modifying existing aircraft with near-axis protuberances. This latter application seems to have been successful in the case of the F-15 exercise.

The ease of inputting information also appears attractive for uses in systems studies. In these types of studies, detailed geometry may not be available, but volume requirements may. This volume information would be well suited as input data for the modified supersonic area rule.

Acknowledgment

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Approximation to the Optimization of a Coast-Glide Trajectory

Israel Shruster* and Israel Carpas†
Israel Aircraft Industries Ltd., Yahud, Israel

Introduction

THE need to achieve maximum range for unpowered gliding flight leads to the problem of finding an angle-of-attack function such that the range is maximized. This Note presents a method for determining an approximation to the angle-of-attack function which is flexible and does not require solving a boundary value problem, as do the usual methods. The method consists of dividing the trajectory into intervals for which the angle of attack assumes a constant value. A relatively small number of intervals provided a close approximation to the optimum.

Formulation of the Approach

The method presented herein provides the angle-of-attack function that is optimal among a specific class. The solution is obtained by dividing the trajectory into a number of intervals, for each of which the angle of attack α assumes a constant value. As the number of intervals increases the solution approaches the optimum (maximum range).

Let u be some parameter that changes monotonically along the trajectory, and let the angle of attack assume discrete values for given n intervals:

$$
\begin{aligned}
\alpha(u) &= C_1 \qquad u_0 \le u < u_1 \\
&= C_2 \qquad u_1 \le u < u_2 \\
&= C_n \qquad u_{n-1} \le u \le u_n
\end{aligned}
$$

We are looking for the constants C_i that maximize the range. The solution of the problem described here can be obtained by well-known optimization algorithms for finding the maximum (or minimum) of functions of several variables. It is easy to see that a trajectory obtained in this way approaches the optimal trajectory as n grows.

An important advantage of this approach is simplicity in building the numerical codes, and the flexibility of the codes

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*Mathematician, Simulation Department, MBT Weapons Systems.
†Aeronautical Engineer, Aerodynamics Department, MBT Weapons Systems. Member AIAA.